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Dynamics of domain walls in ferrimagnets in the field of an arbitrary polarized acoustic wave

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Abstract. We study the drift of a 180° domain wall in a ferrimagnet with two non-equivalent sublattices in the elastic-stress field generated by an acoustic wave. The dependences of the drift velocity on the amplitude and polarization of the acoustic wave have been obtained. The analysis of long-wave and short-wave limits is carried out. The antiferromagnetic and ferromagnetic limits are explored too. The conditions of the drift of a stripe domain structure in ferrimagnets are obtained.

There is continuing interest in the investigation of the dynamic properties of large-scale magnetic inhomogeneities (domain structures, domain walls (DWs), magnetic solitons etc) in magnetically ordered crystals [1], an important subclass being the interaction of magnetic inhomogeneities with the elastic-stress field generated by an acoustic wave [2, 3].

The magnetoelastic interaction, coupling between the magnetic and elastic subsystems of a crystal, results in: the excitation of acoustic and domain oscillations [3, 4], generation, reradiation and scattering of the moving DW sound [5–8] and inversely, oscillatory and drift motion of the DW itself in a field of an external sound wave [2, 9–11].

Through experimental investigations of direct action of a sound on domains and DWs [12–14] the effects of linear oscillations of 180° DWs and their directional motion have been observed. Note that while the experiments were carried out basically on garnet ferrites, practically all theoretical calculations are made for one-sublattice ferromagnetics (FMs) (the exceptions being [11], a two-sublattice weak ferromagnetic (WFM), and [2], an antiferromagnet (AFM) near sublattice reorientation are considered).

Here the influence of an arbitrary polarized sound wave on 180° DWs in ferrimagnets with two nonequivalent magnetic sublattices is theoretically investigated.

Consider an arbitrary polarized sound wave with arbitrary value of a wavevector $\boldsymbol{k}$ propagating perpendicularly to the plane of a DW, the sound wave being an external field, the inverse effect of the magnetic subsystem on the elastic subsystem being neglected.

The nonlinear macroscopic dynamics of a ferrimagnet with two nonequivalent sublattices in the field of a sound wave can be described on the basis of a Lagrange density function L, expressed in terms of the antiferromagnetism unit vector $\boldsymbol{l}$, $\boldsymbol{l}^2 = 1$ [15]:

$$L(l) = M_0^2 \left\{ \frac{\tilde{\alpha}}{2c^2}\dot{l}^2 - \frac{\tilde{\alpha}}{2}(\nabla l)^2 - \frac{\beta_1}{2}l_z^2 - \frac{\beta_2}{2}l_y^2 - \gamma u_{ik}l_i l_k + \frac{\nu}{gM_0}\frac{l_z\dot{l}_x - l_x\dot{l}_z}{1+l_y} \right\} \qquad (1)$$

where the dot denotes a derivative with respect to time, $M_0^2 = (M_1^2 + M_2^2)/2$, M_0 is the length of the sublattice magnetization vector, $c = gM_0\sqrt{\tilde{\alpha}\delta}/2$ is the minimum spin-wave phase velocity, δ and $\tilde{\alpha}$ respectively, are the homogeneous- and inhomogeneous-exchange coupling constants, g is the gyromagnetic ratio, β_1 and β_2 ($\beta_2 > \beta_1 > 0$) are the effective constants of rhombic anisotropy, u_{ik} is the elastic strain tensor, γ is the magnetoelastic constant,

$$\nu = \frac{|M_1 - M_2|}{M_{1,2}} \tag{2}$$

is the parameter of the dynamic ferrimagnet model [15]. The value of this parameter $\nu \gg \sqrt{\beta/\delta} \sim 10^{-2}$–$10^{-1}$ is a validity criterion for this model, as a certain effective FM with fixed length of a vector of a net magnetization $M_S = |M_S| = |M_1 + M_2 + \cdots|$ (where M_i is the sublattice-magnetization vectors). A similar representation, taking into account a strong exchange interaction between sublattices, is usually used when interpreting the experiments on dynamics of nonlinear excitations in ferrimagnets. Near to a point of compensation of the ferrimagnet, where the lengths of vectors of magnetization of sublattices differ insignificantly ($|M_1 - M_2| \ll M_{1,2}$), the representation of ferrimagnet as effective FM becomes inadequate. If the quantity of parameter ν is rather small ($\nu \to 0$), the dynamic properties of the ferrimagnet are close to the properties of an AFM and essentially differ from properties of FM crystals.

Let us parametrize the vector $\boldsymbol{l}$ by angular variables θ and φ so that

$$l_z = \mathrm{i}l_x = \sin\theta \exp(\mathrm{i}\varphi) \qquad l_y = \cos\theta. \tag{3}$$

In terms of these variables one can note a Lagrange function density of the ferrimagnet (1):

$$\begin{aligned} L(\theta,\varphi) = M_0^2\Bigg[&\frac{\tilde{\alpha}}{2c^2}[\dot{\theta}^2 + \dot{\varphi}^2 \sin^2\theta] - \frac{\tilde{\alpha}}{2}[(\nabla\theta)^2 + \sin^2\theta(\nabla\varphi)^2] - \frac{\beta_1}{2}\sin^2\theta^2\varphi - \frac{\beta_2}{2}\cos^2\theta \\ &-\gamma\{\sin 2\theta(u_{zy}\cos\varphi + u_{yz}\sin\varphi) + u_{yy}\cos^2\theta \\ &+2\sin^2\theta(u_{zz}\cos^2\varphi + u_{xx}\sin 2\varphi + u_{xx}\sin^2\varphi)\} + \frac{\nu}{gM_0}\dot{\varphi}(1-\cos\theta)\Bigg] \end{aligned} \tag{4}$$

and a function that takes into account the dynamic stopping of the DWs

$$Q = \frac{\alpha M_0}{2g}\dot{l}^2 = \frac{\alpha M_0}{2g}(\dot{\theta}^2 + \dot{\varphi}^2\sin^2\theta) \tag{5}$$

where α is the dimensionless Gilbert damping constant.

The equations of motion in terms of the angular variables of vector $\boldsymbol{l}$ in view of relaxation summands look like

$$\begin{aligned} &\tilde{\alpha}\nabla(\sin^2\theta(\nabla\varphi)) - \frac{\tilde{\alpha}}{c^2}\frac{\partial}{\partial t}(\dot{\varphi}\sin^2\theta) - \beta_1\sin^2\theta\sin\varphi\cos\varphi \\ &\quad -\frac{\nu}{gM_0}\dot{\theta}\,\sin\theta + \gamma\{\sin^2\theta(u_{zz}\sin 2\varphi - 2u_{xz}\cos 2\varphi - u_{xx}\sin 2\varphi) \\ &\quad +\sin 2\theta(u_{zy}\sin\varphi - u_{yx}\cos\varphi)\} = \frac{\alpha}{gM_0}\dot{\varphi}\sin^2\theta \end{aligned} \tag{6}$$

$$\begin{aligned} &\tilde{\alpha}\left(\Delta\theta - \frac{1}{c^2}\ddot{\theta}\right) + \sin\theta\cos\theta\left[\tilde{\alpha}\left(\frac{1}{c^2}\dot{\varphi}^2 - (\nabla\varphi)^2\right) - \beta_1\sin^2\varphi + \beta_2\right] \\ &\quad +\frac{\nu}{gM_0}\dot{\varphi}\sin\theta - \gamma\{\sin 2\theta(u_{zz}\cos^2\varphi + u_{xz}\sin 2\varphi + u_{xx}\sin^2\varphi - u_{yy}) \\ &\quad +2\cos 2\theta(u_{zy}\cos\varphi + u_{yx}\sin\varphi)\} = \frac{\alpha}{gM_0}\dot{\theta}. \end{aligned} \tag{7}$$

Equations (6) and (7) are a generalization of the equations of motion of FM and AFM nonlinear dynamics in the field of a sound wave. If $M_1 = M_2$ (i.e. $\nu = 0$), then the system (6), (7) describes dynamics of a two-sublattice AFM; if we neglect the noncollinearity of the sublattices (in a formal limit $\delta \to \infty$ ($\nu = 1$)) they become the Landau–Lifshitz–Gilbert equations for a one-sublattice FM with magnetization $M = M_S l$. Taking this into account we carry out the nonlinear dynamics analysis of DWs in the field of a sound wave in both limit cases.

We suggest that the DW, in which the vector $\boldsymbol{l}$ rotates in the plane of XZ, is stable and the distribution of the magnetization is nonuniform along the Y axis. Such a DW corresponds to $\beta_2 > \beta_1 > 0$ [15] and $\theta = \theta_0 = \pi/2$, and the angular variable $\varphi = \varphi_0(y)$ satisfies the following equation

$$\alpha\varphi_0'' - \beta_1 \sin\varphi_0 \cos\varphi_0 = 0. \tag{8}$$

The solution of equation (8) for a static 180° DW, satisfying the boundary conditions $\varphi_0(-\infty) = 0$, $\varphi_0(+\infty) = \pi$ is

$$\varphi_0' = \frac{1}{y_0}\sin\varphi_0 = \frac{1}{y_0}\cosh^{-1}\frac{y}{y_0} \qquad \cos\varphi_0(y) = -\tanh\frac{y}{y_0} \tag{9}$$

where $y_0 = \sqrt{\tilde{\alpha}/\beta_1}$ means a thickness of DW.

We seek the solutions of the equations of motion (6), (7) using the formalism of perturbation theory for solitons [11, 16–18]. We introduce the collective variable $Y(t)$ as the DW centre, the derivative of which defines the instantaneous velocity of the DW $V(t) = \dot{Y}(t)$. Assuming the acoustic-wave amplitude to be small, we present functions $\theta(y, t)$, $\varphi(y, t)$ and $V(t)$ as a expansion into a series on degrees of amplitude, meaning that we are interested only in a forced motion

$$\begin{aligned} \theta(y,t) &= \frac{\pi}{2} + \theta_1(\xi,t) + \theta_2(\xi,t) + \cdots \\ \varphi(y,t) &= \varphi_0(\xi) + \varphi_1(\xi,t) + \varphi_2(\xi,t) + \cdots \\ V &= V_1(t) + V_2(t) + \cdots \end{aligned} \tag{10}$$

where $\xi = y - Y(t)$; subscripts $n = 1, 2, \ldots$ denote the order of the quantity relative to the sound-wave amplitude; the function $\varphi_0(\xi)$, that describes the motion of undistorted DW, has a structure similar to the static solution (9). The functions θ_n and φ_n give the distortions of the DW shape and the excitation of spin waves caused by interaction with the sound wave.

1. Linear oscillations

We substitute the expansions (10) into the system (6), (7) and separate terms of the different orders of smallness. The equation of zero order describes DW at static rest.

Assuming that the monochromatic sound wave propagates perpendicularly to the plane of the DW $\boldsymbol{u} = \mathrm{Re}\{u_0 \exp[\mathrm{i}(k_y y - \omega t)]\}$ the first-order equations are

$$\begin{aligned} \left(\hat{L} + \sigma + \frac{1}{\omega_1^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \frac{\omega_r}{\omega_1^2}\frac{\mathrm{d}}{\mathrm{d}t}\right)\theta_1(\xi,t) - \frac{\omega_v}{\omega_1^2}\frac{\mathrm{d}\varphi_1(\xi,t)}{\mathrm{d}t} &= -\frac{1}{y_0\omega_1^2}\omega_v V_1 \sin\varphi_0 \\ &+ \frac{\mathrm{i}k\gamma}{\beta_1}\exp[\mathrm{i}(k(Y+\xi) - \omega t)](u_{0x}\sin\varphi_0 + u_{0z}\cos\varphi_0) \end{aligned} \tag{11}$$

$$\left(\hat{L} + \frac{1}{\omega_1^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \frac{\omega_r}{\omega_1^2}\frac{\mathrm{d}}{\mathrm{d}t}\right)\varphi_1(\xi,t) + \frac{\omega_v}{\omega_1^2}\frac{\mathrm{d}\theta_1(\xi,t)}{\mathrm{d}t} = -\frac{1}{y_0\omega_1^2}(\dot{V}_1 + \omega_r V_1)\sin\varphi_0 \tag{12}$$

where $\omega_1 = c/y_0 = gM_0\sqrt{\beta\delta}/2$, $\omega_r = \alpha\delta g M_0/4$, $\omega_v = \nu\delta f M_0/4$ are frequency characteristics of the material, $k = k_y = \omega/s$ is the sound wavevector, ω and s are the frequency and velocity of the wave respectively and $\sigma = (\beta_2 - \beta_1)/\beta_1$. Operator

$$\hat{L} = -y_0\frac{\mathrm{d}^2}{\mathrm{d}\xi^2} + 1 - \frac{2}{\cosh^2(\xi/y_0)} \tag{13}$$

has known normalized wavefunctions and a spectrum [19]

$$f_0(\xi) = \frac{1}{\sqrt{2y_0}} \cosh^{-1} \frac{\xi}{y_0} \qquad \lambda_0 = 0$$
$$f_p(\xi) = \frac{1}{b_p\sqrt{L}} \left(\tanh \frac{\xi}{y_0} - \mathrm{i} p y_0 \right) \exp(\mathrm{i}p\xi) \qquad \lambda_p = 1 + p^2 y_0^2 \tag{14}$$

where $b_p = \sqrt{1 + p^2 y_0^2}$, L is the length of the crystal.

We seek the solution to the system of equations of the first approximation (11), (12) as an expansion over a complete orthonormal set of eigenfunctions$\{f_0(\xi),\ f_k(\xi)\}$

$$\theta_1(\xi, t) = \mathrm{Re}\left\{ \sum_p [c_p^{(1)} f_p(\xi) + c_0^{(1)} f_0(\xi)] \exp[\mathrm{i}(ky - \omega t)] \right\} \tag{15}$$

$$\varphi_1(\xi, t) = \mathrm{Re}\left\{ \sum_p [d_p^{(1)} f_p(\xi) + d_0^{(1)} f_0(\xi)] \exp[\mathrm{i}(ky - \omega t)] \right\}. \tag{16}$$

We should note that $d_0^{(1)}$ in the expansion (16) corresponds to the shear (Goldstone) mode. However, when introducing the collective coordinate $Y(t)$ in the definition of ξ this degree of freedom has already been taken into account. Therefore it is necessary to put $d_0^{(1)} = 0$ in the expansion (16).

Using a standard procedure of definition of coefficients in expansions (15), (16) we obtain the solutions of the first-order equations (11), (12) as

$$\varphi_1(\xi, t) = \frac{\gamma \pi k^2 y_0}{2\beta_i} \mathrm{Re}\{B_1(\xi) \exp[\mathrm{i}(kY - \omega t)]\} \tag{17}$$

$$\theta_1(\xi, t) = \frac{\gamma \pi k^2 y_0}{2\beta_i} \mathrm{Re}\{B_2(\xi) \exp[\mathrm{i}(kY - \omega t)]\} + \mathrm{Re}\{A(\xi) \exp[\mathrm{i}(kY - \omega t)]\}. \tag{18}$$

Here we have introduced the notations

$$A = \frac{1}{y_0 \omega_1^2} \frac{\mathrm{i} q_3 \lfloor \dot{V}_1 + \omega_r V_1 \rfloor - q \omega_v V_1}{(\sigma - q) q + q_3^2} \sin \varphi_0$$
$$B_1(\xi) = -\frac{y_0}{\pi} q_3 [u_{0z} D_1(\xi) + \mathrm{i} u_{0x} D_2(\xi)] + q_3 b_1 f_k(\xi)$$
$$B_2(\xi) = -\frac{y_0}{\pi} [u_{0z} D_3(\xi) + \mathrm{i} u_{0z} D_4(\xi)] + b_2 f_k(\xi) + b_3 \sin \varphi_0$$
$$D_1(\xi) = \sqrt{L} \int_{-\infty}^{\infty} \mathrm{d}p\ f_p(\xi) \frac{\Omega(p, q)}{\sinh[\pi y_0 (k - p)/2]}$$
$$D_2(\xi) = \sqrt{L} \int_{-\infty}^{\infty} \mathrm{d}p\ f_p(\xi) \frac{\Omega(p, q)}{\cosh[\pi y_0 (k - p)/2]}$$
$$D_3(\xi) = \sqrt{L} \int_{-\infty}^{\infty} \mathrm{d}p\ f_p(\xi) \frac{\Omega(p, q)(\lambda_p - q)}{\cosh[\pi y_0 (k - p)/2]}$$
$$D_4(\xi) = \sqrt{L} \int_{-\infty}^{\infty} \mathrm{d}p\ f_p(\xi) \frac{\Omega(p, q)(\lambda_p - q)}{\sinh[\pi y_0 (p - k)/2]}$$
$$b_1 = \frac{2\sqrt{L} u_{0z} \Omega(k, q)}{\pi k y_0} \qquad b_2 = -\mathrm{i}(\lambda_k - q) b_1$$
$$b_3 = \frac{q}{(\sigma - q) q + q_3^2} \left[\frac{u_{0z}}{\cosh(\pi k y_0/2)} + \frac{\mathrm{i} u_{0x}}{\sinh(\pi k y_0/2)} \right] \tag{19}$$

where

$$\Omega(p,q) = \{b_p[(\lambda_p - 1)(\lambda_p - 1 + \sigma) = q_3^2]\}^{-1} \qquad q = q_1 + \mathrm{i}q_2 \qquad q_1 = \left(\frac{\omega}{\omega_1}\right)^2$$
$$q_2 = \left(\frac{\omega\omega_r}{\omega_1^2}\right) \qquad q_3 = \left(\frac{\omega\omega_v}{\omega_1^2}\right).$$

From (17)–(19) the acoustic wave propagates perpendicularly to rotation planes of a vector $\boldsymbol{l}$ in the DW, exciting the localized and non-localized spin waves. And these excitations are provided only with transversal acouctic oscillations.

Proceeding from the requirement of vanishing of the Goldstone mode amplitude ($d_0^{(1)} = 0$), that is equivalent to a requirement of orthogonality of the right side of the equation (12) to eigenfunction $f_0(\xi)$, we obtain the equation for definition of the velocity of the DW

$$\dot{V}_1(t) + \left(\omega_r - \frac{\mathrm{i}q_3\omega_v}{\sigma - q}\right) V_1(t) = \frac{\pi\gamma(ky_0)^2 q_3\omega_1^2}{2\beta_1(\sigma - q)} \left[\frac{u_{0x}}{\sinh(\pi k y_0/2)} - \frac{\mathrm{i}u_{0z}}{\cosh(\pi k y_0/2)}\right] \times \exp[\mathrm{i}(kY - \omega t)]. \tag{20}$$

The equation (20) is a generalization of the corresponding equations for a one-sublattice FM and two-sublattice AFM. Its solution is

$$V_1(t) = \frac{\gamma q_3(ky_0)^2\pi\omega_1^2}{2\beta_1(\sigma - q)(\omega_r - \mathrm{i}\omega - \mathrm{i}q_3\omega_v/(\sigma - q))} \left[\frac{u_{0x}}{\sinh(\pi k y_0/2)} - \frac{\mathrm{i}u_{0z}}{\cosh(\pi k y_0/2)}\right] \times \exp[\mathrm{i}(kY - \omega t)]. \tag{21}$$

Thus to first order of perturbation theory the DW velocity is $V_1(t) \sim \nu$. This means that in the AFM limit $V_1^{AFM}(t) = 0$, i.e. in the geometry of the problem considered, to first order the sound wave does not generate oscillations of DWs in a two-sublattice AFM. The similar situation also takes place in the WFM [11]. In the FM limit the equation (20) is reduced to

$$\dot{V}_1(t) + \omega_v\left(\alpha - \frac{\mathrm{i}q_3}{\sigma - \mathrm{i}\alpha q_3}\right) V_1(t) = \frac{\pi\gamma\omega_0^2 q_3\delta(ky_0)^2}{8\beta_1^2(\sigma - \mathrm{i}\alpha q_3)} \left[\frac{u_{0x}}{\sinh(\pi k y_0/2)} - \frac{\mathrm{i}u_{0z}}{\cosh(\pi k y_0/2)}\right] \times \exp[\mathrm{i}(kY - \omega t)] \tag{22}$$

where $\omega_0 = \beta_1 g M_0$. This equation has the solution

$$V_1(t) = \frac{\gamma q_3\delta(ky_0)^2\pi\omega_0^2}{8\beta_1^2(\sigma - \mathrm{i}\alpha q_3)(\alpha\omega_v - \mathrm{i}\omega - \mathrm{i}q_3\omega_v/(\sigma - \mathrm{i}\alpha q_3))} \left[\frac{u_{0x}}{\sinh(\pi k y_0/2)} - \frac{\mathrm{i}u_{0z}}{\cosh(\pi k y_0/2)}\right] \times \exp[\mathrm{i}(kY - \omega t)]. \tag{23}$$

Using expressions (21) and (23) it is possible to find a displacement of the DW $y(t)$ for the frequency period:

$$y(t) = \frac{\pi\gamma(ky_0)^2\omega_v}{2\beta_i[Q_1^2 + Q_2^2]}\left[\left(\frac{Q_1 u_{0x}}{\sinh(\pi k y_0/2)} + \frac{Q_2 u_{0z}}{\cosh(\pi k y_0/2)}\right)\sin(kY - \omega t) + \left(\frac{Q_2 u_{0x}}{\sinh(\pi k y_0/2)} - \frac{Q_1 u_{0z}}{\cosh(\pi k y_0/2)}\right)\cos(kY - \omega t)\right] \tag{24}$$

where in the FM limit $Q_1 = \alpha\sigma\omega_v$, and $Q_2 = q_3\omega_v(1 - \alpha^2)$; in the case of a ferrimagnet $Q_1 = \omega_r(\sigma - 2q_1)$, $Q_2 = \omega_v q_3 + \omega(\sigma - q_1) - \omega_r q_2$. In particular for gadolinium garnet ($Gd_3Fe_5O_{12}$) the mean value of the displacement modulus at frequency $\omega \sim 10^6$ s^{-1} at $\nu = 2 \times 10^{-2}$ and $\nu = 2 \times 10^{-3}$ is 10^{-8} cm is 10^{-9} cm, respectively. In the FM limit at the same frequency the displacement is 10^{-6} cm. The obtained mean values of the modulus of displacement agree well with [20].

2. Nonlinear motion of DW

The second-order equations for functions $\theta_2(\xi, t)$ and $\varphi_2(\xi, t)$ are

$$\hat{L}\varphi_2 = -\frac{\mathrm{i}k\gamma}{\beta_1}\exp[\mathrm{i}(k(Y+\xi)-\omega t)]\theta_1(u_{0z}\sin\varphi_0 - u_{0x}\cos\varphi_0) - 2y_0\theta_1\theta_1'\sin\varphi_0 + \frac{1}{\omega_1^2}(\dot{V}_1+\omega_r V_1)\varphi_1' + \frac{1}{y_0\omega_1^2}(\dot{V}_2+\omega_r V_2)\sin\varphi_0 + \varphi_1^2\sin 2\varphi_0 + \frac{\omega_v}{\omega_1^2}V_1\theta_1' - \frac{1}{\omega_1^2 y_0^2}V_1^2\sin\varphi_0\cos\varphi_0 \tag{25}$$

$$(\hat{L}+\sigma)\theta_2 = -\frac{\mathrm{i}k\gamma}{\beta_1}\exp[\mathrm{i}(k(Y+\xi)-\omega t)][2\theta_1 u_{0y} + \varphi_1(u_{0z}\sin\varphi_0 - u_{0x}\cos\varphi_0)] + \frac{1}{\omega_1^2}(\dot{V}_1+\omega_r V_1)\theta_1' + 2y_0\theta_1\varphi_1'\sin\varphi_0 + \theta_1\varphi_1\sin 2\varphi_0 - \frac{\omega_v}{\omega_1^2}\left(V_1\varphi_1' + \frac{1}{y_0}V_2\sin\varphi_0\right). \tag{26}$$

One can seek solutions of these equations, here as in (11), (12), as an expansion in the eigenfunctions of the operator $\hat{L}$ (15), (16).

A complete solution of system (25), (26) is not attempted here but we present only the DW velocity in the second order of perturbation theory $V_2(t)$. To determine the velocity $V_2(t)$ it is enough to find the coefficient $d_0^{(2)}$ describing the Goldstone mode in the expansion

$$\varphi_2(\xi, t) = \mathrm{Re}\left\{\sum_p [d_p^{(2)} f_p(\xi) + d_2^{(2)} f_0(\xi)]\exp[\mathrm{i}(ky-\omega t)]\right\} \tag{27}$$

and to equate it to zero (i.e. $d_0^{(2)} = 0$). Then by using (17)–(19) the equations for $V_2(t)$ can be written as

$$\dot{V}_2(t) + \omega_r V_2 = N + N_1\exp(2\mathrm{i}\omega t) + N_2\exp(-2\mathrm{i}\omega t) \tag{28}$$

where

$$N = \mathrm{Re}\left[\int_{-\infty}^{\infty}\mathrm{d}\xi\,\sin\varphi_0\left\{\frac{\mathrm{i}k\gamma\omega_1^2}{2\beta_1}\exp[\mathrm{i}(k(Y+\xi)-\omega t)]\theta_1(\xi,t)(u_{0z}\sin\varphi_0 - u_{0x}\cos\varphi_0) - V_1\frac{\omega_v}{2}\theta_1'(\xi,t) + \omega_1^2 y_0\theta_1(\xi,t)\theta_1'(\xi,t)\sin\varphi_0 - \frac{1}{2}(\dot{V}_1+\omega_r V_1)\varphi_1'(\xi,t) - \omega_1^2\varphi_1^2(\xi,t)\sin\varphi_0\cos\varphi_0\right\}\right]. \tag{29}$$

The explicit forms of expressions for N_1 and N_2 which are similar to (29) are not required here because N_1 and N_2 vanish after further time averaging the solution of equation (28).

After integrating equation (28) and averaging over the period of oscillations of the obtained result the DW drift velocity in the field of an acoustic wave becomes

$$V_{dr} = \overline{V_2(t)} = \mu_{xx}(k)(ku_{0,x})^2 + \mu_{xz}(k)(ku_{0x})(ku_{0z}) + \mu_{zz}(k)(ku_{0z})^2 \tag{30}$$

where $\mu_{ij}(k)$ are nonlinear mobility (NM) DWs in the field of an acoustic wave.

Since the analysis of the general expressions for $\mu_{ij}(k)$ is not productive we consider the asymptotic cases.

In the *long-wave* approximation (that is the most interesting from the point of view of experiment ($ky_0 \ll 1$)) corresponding to the frequency range $\omega = sk \ll 10^{10}$ s^{-1}, the expressions for NM are reduced to

$$\mu_{xz}(k) = \mu_0[A(k) + F(k)]$$
$$\mu_{xx}(k) = \mu_0 \frac{3ky_0q_2}{(\sigma+1)^2\sigma^2} \qquad \mu_{zz}(k) = -\mu_0 \frac{4ky_0q_2}{(\sigma+1)^2\sigma^2} \tag{31}$$

where

$$\mu_0 = \nu_0 \frac{\delta(\gamma M_0)^2}{4\beta_1} \qquad \nu_0 = \frac{\pi y_0 g^2}{4\omega_r} \qquad A(k) = -\frac{2(ky_0)^2}{\sigma^2(\sigma+1)^2}$$
$$F(k) = -\frac{q_3^4}{2(\sigma+1)^2(\sigma^2\alpha^2+q_3^2)}.$$

Since $\mu_{xx} \sim \mu_{zz} \sim q_2 \ll ky_0 \ll 1$, it is obvious that the principal contribution to the DW drift velocity (30) gives $\mu_{xz}(k)$. The structure of expressions (31) underlines the fact that the NM of a ferrimagnet is formed by terms of ferromagnetic $F(k)$ and antiferromagnetic $A(k)$ origin, each of which provides the NM in the corresponding limit.

Indeed, in the AFM-limit ($\nu \to 0$) the second term on the right side of expression (31) becomes zero and the explicit view NM $\nu_{ij}^{AFM}(k)$ is obtained

$$\mu_{xz}^{AFM}(k) = -\mu_0 \frac{2(ky_0)^2}{\sigma^2(\sigma+1)^2}$$
$$\mu_{xx}^{AFM}(k) = \mu_0 \frac{q_2ky_0}{10\sigma^2} \qquad \mu_{zz}^{AFM}(k) = \mu_0 \frac{q_2ky_0}{3\sigma^2}. \tag{32}$$

The expressions (32) coincide, apart from notational differences, with the corresponding expressions for the WFM [11], if we neglect in the latter the Dzyaloshinskii interaction. The frequency dependence of NM $\mu_{xz}^{AFM}(k)$ in the long-wave approximation is shown in figure 1(a).

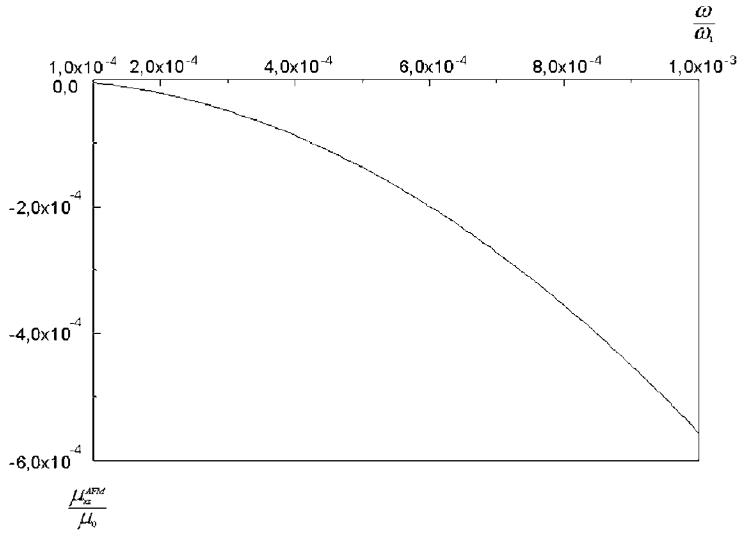
In the FM limit ($\delta \to \infty$, $\nu \to 1$) the first term in expression (31) becomes zero and the NM $\mu_{ij}^{FM}(k)$ becomes

$$\mu_{xz}^{FM}(k) = -\mu_0 \frac{q_3^4}{2(\sigma+1)^2(\sigma^2\alpha^2+q_3^2)}$$
$$\mu_{xx}^{FM}(k) = \mu_0 \frac{5ky_0q_2}{(\sigma+1)^2(\sigma^2\alpha^2+q_3^2)} \qquad \mu_{zz}^{FM}(k) = -\mu_0 \frac{3ky_0q_2q_3^2}{4(\sigma+1)(\sigma^2\alpha^2+q_3^2)}. \tag{33}$$

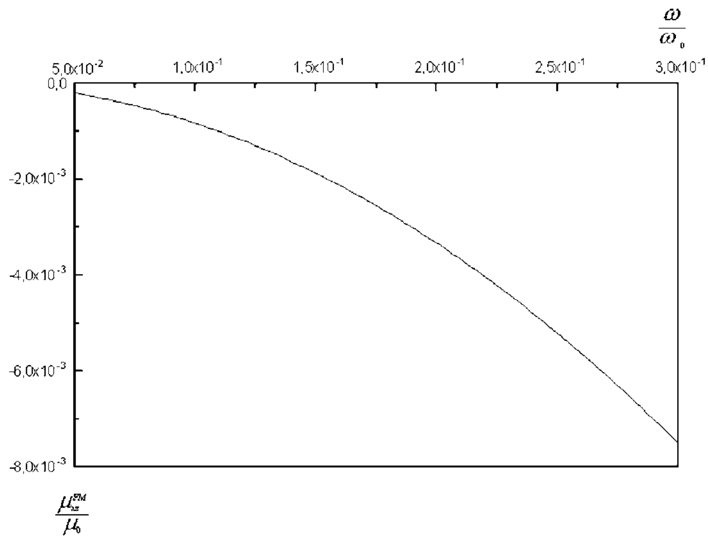
The expression for the drift velocity (30) using $\mu_{ij}^{FM}(k)$ from (33) is in qualitative agreement with the corresponding expression obtained in [10] directly for the one-sublattice FM. There cannot be a complete quantitative coincidence, since in [10] the approximate equations of Slonczewski and the slightly different conditions of deriving the required velocity are used. The frequency dependence of NM $\mu_{xz}^{FM}(k)$ in the long-wave approximation is shown in figure 1(b).

One can see from figure 1(a) and 1(b) that the nonlinear mobility of the DW, as well as the drift velocity (30), monotonically grow with increasing sound wave frequency both in the AFM and FM limits.

Let us give the numerical estimations of nonlinear mobility and drift velocity of the DW. In the AFM limit we use the numerical values of parameters characterizing a ferrimagnet near the compensation point (for example, $Gd_3Fe_5O_{12}$ [21]): $y_0 \approx 10^{-4}$ cm, $M_0 = 10$ Oe, $\gamma M_0^2 \sim 10^7$ erg cm^{-3}, $g = 1.76 \times 10^7$ (s Oe)$^{-1}$, $\omega_1 \sim 10^{11}$ s^{-1}, $\omega_r \sim 10^8$ s^{-1}. These values of parameters correspond to $\mu_0 = 0.9 \times 10^{14}$ cm s^{-1}. At the typical sound velocity $s \sim 10^5$ cm s^{-1} and the maximum admissible value of the strain tensor $ku_0 \sim 10^{-5}$ the DW drift velocity in the AFM limit is equal to 10 cm s^{-1}.



(a)

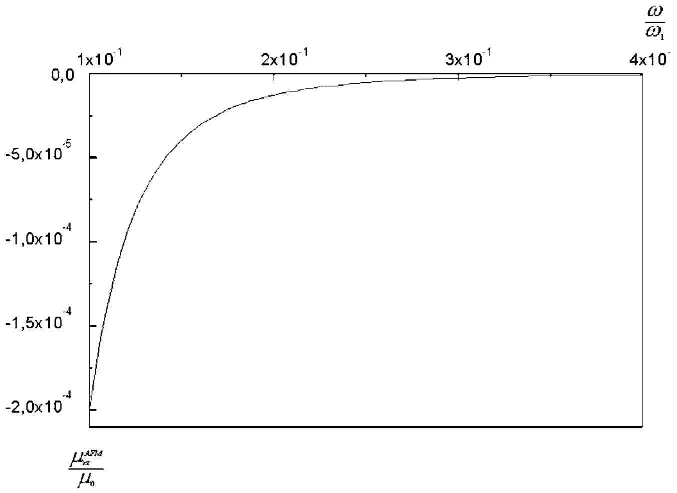


(b)

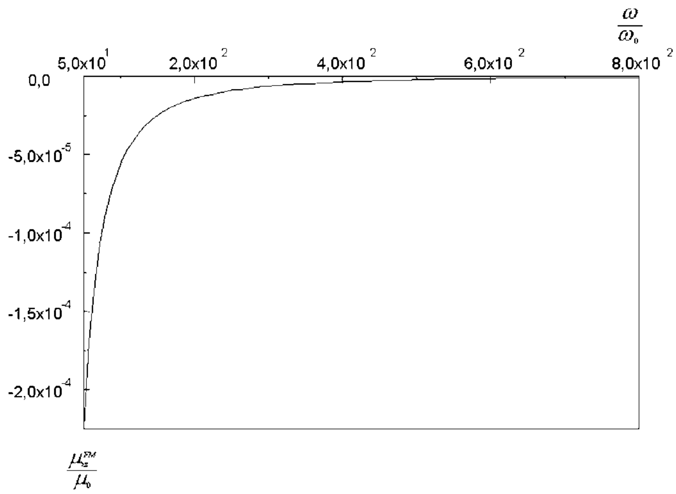
Figure 1. (a) Frequency dependence of the second-order mobility μ_{xz}^{AFM} in the antiferromagnetic limit ($ky_0 \ll 1$). (b) Frequency dependence of the second-order mobility μ_{zz}^{FM} in the ferromagnetic limit ($ky_0 \ll 1$).

In the FM limit we use the typical values of parameters of a ferrimagnet far from the compensation point (for example, (YFe_5O)) [21]: $y_0 \approx 5 \times 10^{-5}$ cm, $M_0 = 140$ Oe, $\gamma M_0^2 \sim 10^7$ erg cm^{-3}, $g = 1.76 \times 10^7$ (s Oe)$^{-1}$, $\omega_0 \approx 3 \times 10^9$ s^{-1}, $\omega_r \sim 10^{10}$ s^{-1}. These values of parameters correspond to $\mu_0 = 0.3 \times 10^{12}$ cm s^{-1}. At the same parameters of the acoustic wave the drift velocity is 0.3 cm s^{-1}.

The drift velocity in ferrimagnet $(Y, Sm, Ca)_3(Fe, Ge)_5O_{12}$ [21] at $\nu \sim 10^{-2}$–10^{-3} is 1 cm s^{-1}.



(a)



(b)

Figure 2. (a) Frequency dependence of the second-order mobility μ_{xz}^{AFM} in the antiferromagnetic limit ($ky_0 \ll 1$). (b) Frequency dependence of the second-order mobility μ_{xz}^{FM} in the ferromagnetic limit ($ky_0 \ll 1$).

In the *short-wave* approximation ($ky_0 \gg 1$), corresponding to the hypersonic frequencies $\omega \gg 10^{11}\ \mathrm{s}^{-1}$, the nonlinear mobilities of the ferrimagnet decrease as ω^{-4}:

$$\mu_{xz} = -\mu_0 \frac{2}{(ky_0)^4} \qquad \mu_{xx} = -\mu_0 \frac{q_2}{ky_0 q_1^2} \qquad \mu_{zz} = -\mu_0 \frac{4q_2}{\pi k y_0 q_1^2} \tag{34}$$

and the value of the effect in this case is inversely proportional to the square of the frequency, which agrees with [22].

The expressions for mobilities in the corresponding limits are

$$\text{AFM limit: } \mu_{xz} = -\mu_0 \frac{2}{(ky_0)^4} \qquad \mu_{xx} = -\mu_0 \frac{2q_2}{5(ky_0)^5} \qquad \mu_{zz} = -\mu_0 \frac{2q_2}{\pi (ky_0)^5} \tag{35}$$

FM limit: $\mu_{xz} = -\mu_0 \frac{1}{q_3^2}$ $\mu_{xx} = \frac{\mu_0 2q_2}{2(ky_0)^5}$ $\mu_{zz} = -\mu_0 \frac{2\alpha}{\pi(ky_0)q_3}$. (36)

The frequency dependence for maximal NM $\mu_{xz}(k)$ in the short-wave approximation is shown in figures 2(a) and (b) for the AFM and FM limits, respectively.

3. Drift of a stripe-domain structure

We now consider the possibility of drift in a sound-wave field of a stripe domain structure (SDS) consisting of 180° DWs. It is necessary to keep in mind that neighbouring DWs in such a structure have opposite topological charges, which are determined by the boundary conditions to the equation (8). Besides the rotation of vector $\boldsymbol{l}$ in various DWs can be about the positive or the negative direction of the Z axis. These two factors determine the DW drift direction in a field of fixed frequency ω. SDS drift is possible only if neighbouring DWs move in the same direction.

We define the topological DW charge $R = \pm 1$ and the parameter $\rho = \pm 1$, which describes the direction of rotation of vector $\boldsymbol{l}$ in a DW, as follows: $l_z(y = \pm\infty) = \mp R, l_x(y = 0) = \pm\rho$.

The DWs considered in the previous sections, with distribution of magnetization (9) correspond to $R = \rho = +1$. Generally, instead of (9) we obtain

$$\varphi_0' = \frac{1}{y_0} R \sin\varphi_0 = \frac{1}{y_0} R\rho \cosh^{-1}\frac{y}{y_0} \qquad \cos\varphi_0(y) = -R \tanh\frac{y}{y_0}. \tag{37}$$

The analysis shows that in the general case the drift velocity of a DW, for a ferrimagnet and its two extreme cases, with given values of parameters R and ρ, is presented as:

$$V_{dr} = \mu_{xx}(k)(ku_{0x})^2 + R\rho\mu_{xz}(k)(ku_{0x})(ku_{0z}) + \mu_{zz}(k)(ku_{0z})^2. \tag{38}$$

Since the dominant contribution to the drift velocity is provided by the off-diagonal mobility μ_{xz}, for the corresponding term in (38) to be the same for all DWs and for all the DWs in the structure to drift in the same direction, it is necessary that the parameters ρ in the neighbouring DWs as well as the topological charges R, must be unlike, i.e. the rotation of a vector $\boldsymbol{l}$ in the neighbouring DWs should be in the same direction. The similar situation (the possibility of drift of the DS in the field of the acoustic wave) was observed in a WFM in [11].

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